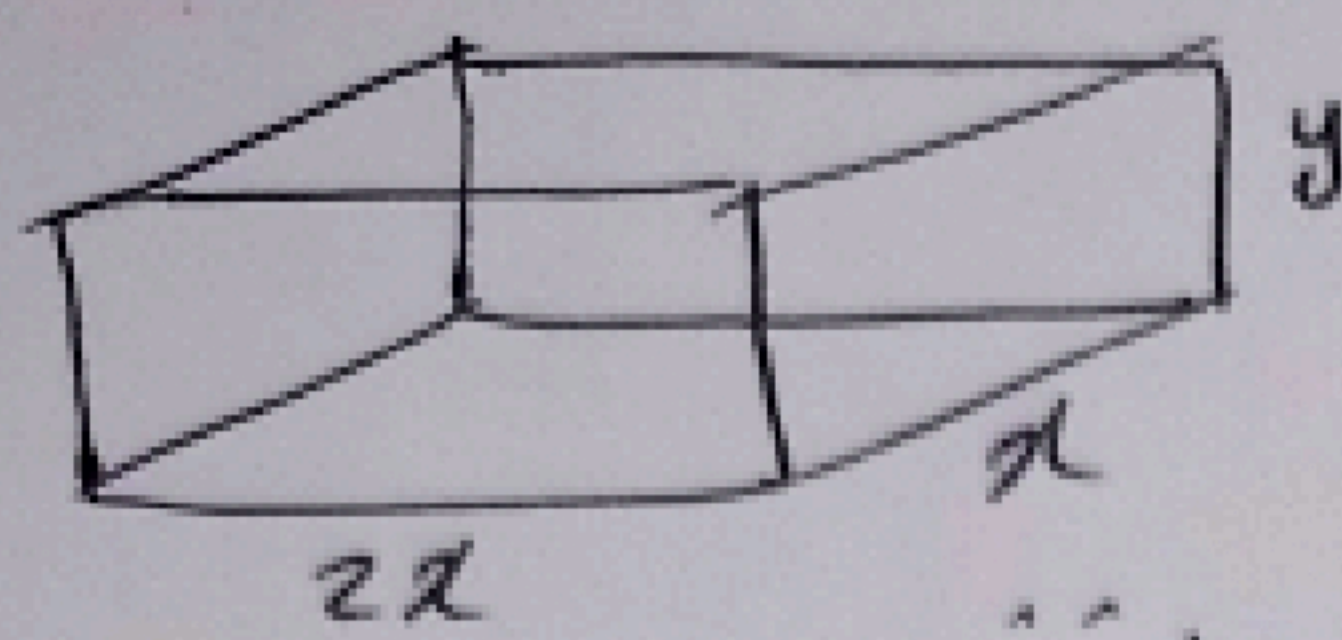


أ. إيجاد زحل

(4)

4
ب

مساحة السطح = مساحة الجوانب + مساحة القاعدة



مساحة السطح = مساحة الجوانب + مساحة القاعدة
 $2x \times x + 2x \times y + 2x \times y + x \times y + x \times y = A$
 $2x^2 + y \times 6x = A$

$2400 = 2x^2 + 6xy \quad / \quad 6x$

$\frac{2400}{6x} = \frac{2x^2}{6x} + y$

$\frac{400}{x} - \frac{x}{3} = y$

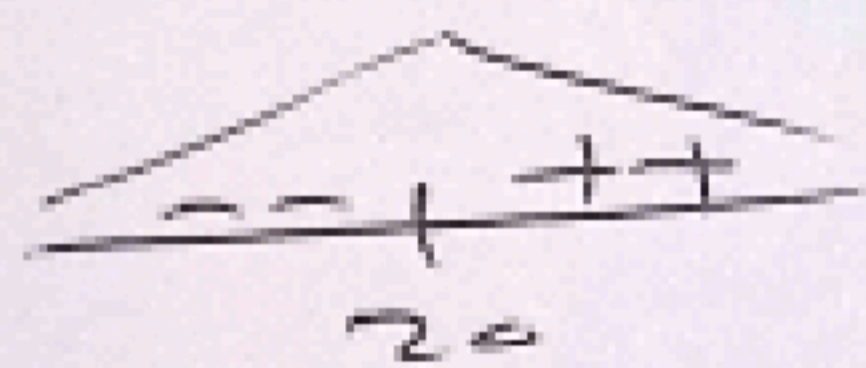
$V = 2x \cdot x \cdot y = 2x^2 \cdot y$

$V = 2x^2 \left(\frac{400}{x} - \frac{x}{3} \right)$

$V = 800x - \frac{2x^3}{3}$

$V' = 800 - 2x^2 = 0$

$x^2 = 400 \implies x = 20$



$y = \frac{400}{20} - \frac{20}{3} = \frac{20}{1} - \frac{20}{3}$

$y = \frac{40}{3}$

$2x = 40$ الطول

5
ب

$z = -2 - i\sqrt{12}$

$z = r(\cos \theta + i \sin \theta)$

$r = |z| = \sqrt{4 + 12} = 4$

العدد المركب يقع في الربع الثالث
 $a = -2$
 $b = -\sqrt{12}$

$\theta = -(\pi - \tan^{-1} \frac{\sqrt{3}}{2})$

$\theta = -(\pi - \frac{\pi}{3}) = -(\frac{2\pi}{3})$

$\therefore z = 4 \left(\cos \frac{-2\pi}{3} + i \sin \frac{-2\pi}{3} \right)$

(b) $z^3 + 3z^2 - 5z - 39 = 0$

بالجربة $z = 3$ أحد الجذور

$27 + 27 - 15 - 39 = 0$

$\therefore z - 3$ أحد العوامل

لإيجاد العوامل الرئيسية، نقسم

z^3	z^2	z^1	z^0
	3	-5	-39
$\boxed{3}$ +	0	3	18
			39
	1	6	13
			0

$z^2 + 6z + 13$

هو المعامل الأخير

$\Delta = b^2 - 4ac = 36 - 4(1)(13) = 36 - 52 = -16$

$z = \frac{-6 \pm \sqrt{-16}}{2}$

$z = \frac{-6 \pm 4i}{2} = -3 + 2i$

$= -3 - 2i$

الجذور هي $3, -3 + 2i, -3 - 2i$

(c) المنطقة الظللة هي تقاطع

$|z - (2 + 2i)| = 2$ دائرة

$|z - (2 + 2i)| \leq 2$ أو

والمنطقة المصغرة. (منطقة)

$\text{Arg}(z - (2 + 2i)) = \frac{\pi}{2}$

$\text{Arg}(z - (2 + 2i)) = \frac{\pi}{4}$ و

$\tan \theta = \frac{2}{2} = 1$ حيث

$\frac{\pi}{4} \leq \text{Arg}(z - (2 + 2i)) < \frac{\pi}{2}$

بالنسبة للمنطقة

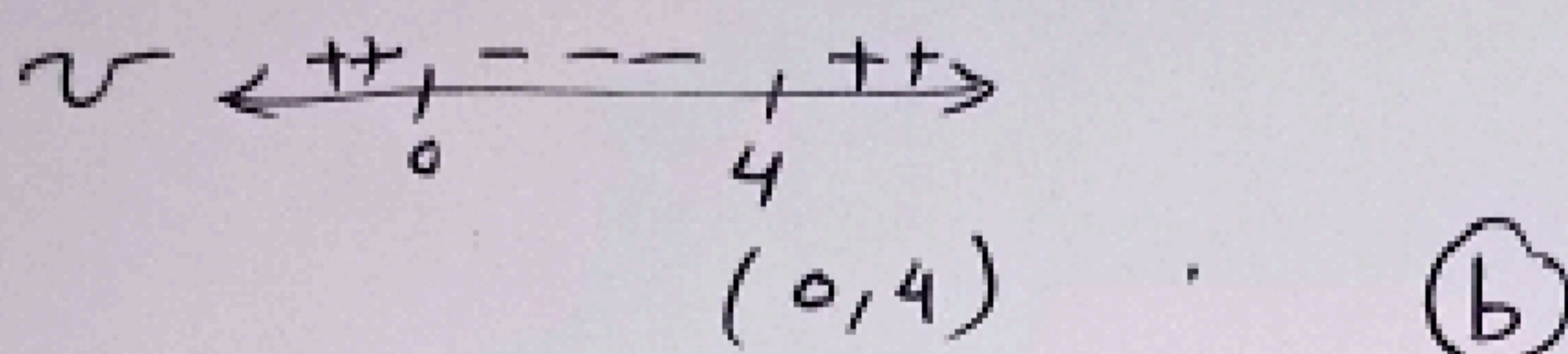
$\frac{\pi}{4} \leq \text{Arg}(z - (2 + 2i)) < \frac{\pi}{2}$

أ. كاد زحرابه

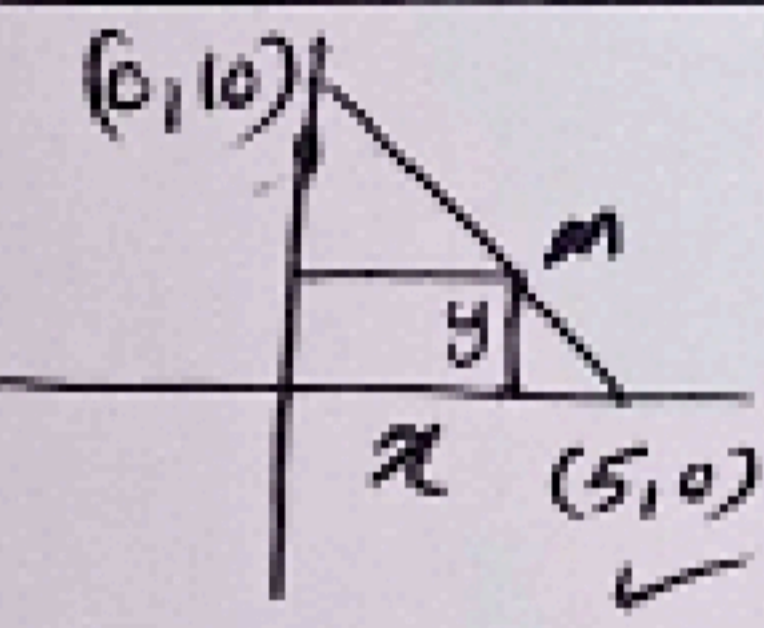
(2)

(14) f صغر لـ f' عندما $0 < f' < 1$ و f' متزايدة
 $[2, 4]$ ∴ (d)

(15) $v < 0$
 $3t^2 - 12t < 0$
 $3t^2 - 12t = 0 \Rightarrow 3t(t-4)$
 $t = 0, 4$



(16) $R' = 0 \Rightarrow -100x + 600 = 0$
 $100x = 600 \Rightarrow x = 6$
 $R(6) = -50 \cdot 36 + 200(178)$
 $= -1800 + 35600$
 $= 33800$ (c)

(17) 
 معادلة الخط
 $y - 0 = \frac{10 - 0}{0 - 5}(x - 5)$
 $y = -2(x - 5)$
 $y = -2x + 10$

$A = x \cdot y = x(-2x + 10)$
 $A = -2x^2 + 10x$
 $A' = -4x + 10 = 0 \Rightarrow 10 = 4x$
 $x = 2.5 = \frac{5}{2}$ (d)

(18) $2021 \cdot i \times \sqrt{-4} = i \times i \cdot 2$
 $= 2i^2 = -2$ (b)

(19) $3x + 3y + 12xi - 4y = 43 + (32 - y)i$
 $(3x + 3y) + (12x - 4y)i = 43 + (32 - y)i$
 $3x + 3y = 43 \Rightarrow 3x + 3y = 43 +$
 $12x - 4y = 32 - y \Rightarrow 12x - 3y = 32$
 $15x = 75$ (b)
 $x = 5$

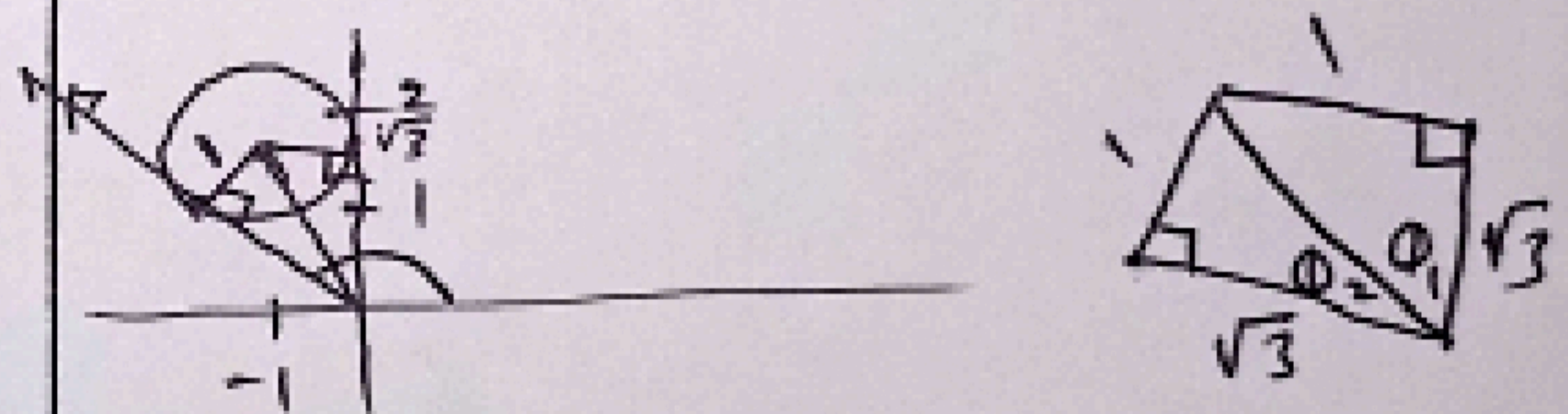
(20) $3 = \sqrt{\frac{q}{k^2} + 8} \Rightarrow q = \frac{q}{k^2} + 8$
 $1 = \frac{q}{k^2} \Rightarrow k = 3$ (c)

(21) $\frac{a^2 + b^2}{a + bi} \times \frac{a - bi}{a - bi} = 2 + 3i$
 $\frac{(a^2 + b^2)(a - bi)}{a^2 + b^2} = 2 + 3i$
 $a = 2, b = -3$
 $a \times b = -6$ (d)

(22) $\frac{z_1}{z_2} = z \left(\cos \frac{14\pi}{18} + i \sin \frac{14\pi}{18} \right)$ (b)
 $= z \left(\cos \frac{7\pi}{9} + i \sin \frac{7\pi}{9} \right)$

(23) $\sqrt{-7 - 24i} = a + 4i$
 $-7 - 24i = a^2 + 2a \cdot 4i + -16$
 $-7 - 24i = a^2 - 16 + 8ai$
 $-7 = a^2 - 16, -24 = 8a$
 $9 = a^2, a = -3$
 $a = \pm 3$
 $\therefore a = -3$ (a)

(24) $|z - (-1 + i\sqrt{3})| = 1$
 $|z - (a + ib)| = r$
 دائرة مركزها $(-1, \sqrt{3})$ ونصف قطرها 1



$\tan \phi_1 = \tan \phi_2 = \frac{1}{\sqrt{3}} \Rightarrow$
 $\phi_1 = \phi_2 = \frac{\pi}{6}$

$\frac{\pi}{6} + \frac{\pi}{6} + \frac{\pi}{2} =$ الزاوية
 $= \frac{5\pi}{6}$ (a)

(25) أطراف المثلث
 $(-1, 2)$ و $(3, 4)$
 $|z - (-1 + 2i)| = |z - (3 + 4i)|$ (a)

اجابات امتحان الرياضيات العلمي 2023 الفصل الاول

أ. اتحاد زهران

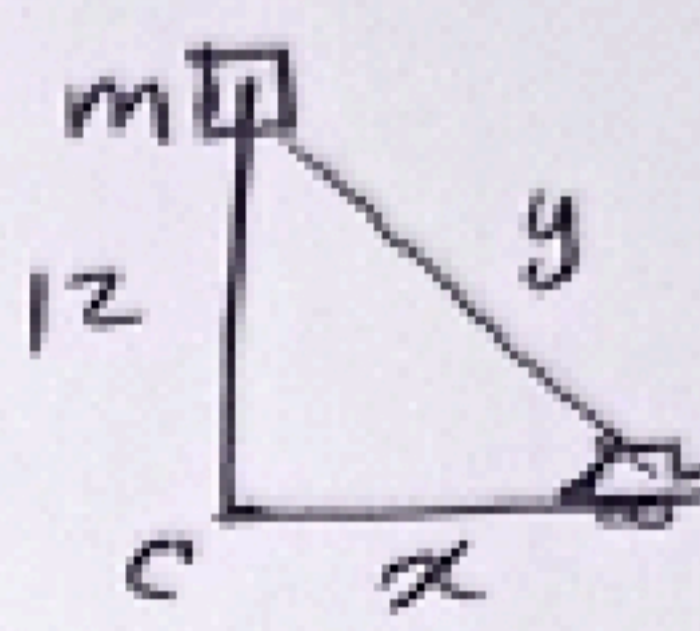
أنموذج A

1

1
 ① $f(x) = e^{\sqrt[3]{x}} \Rightarrow f'(x) = e^{\sqrt[3]{x}} \cdot \frac{1}{3\sqrt[3]{x^2}}$ (d)

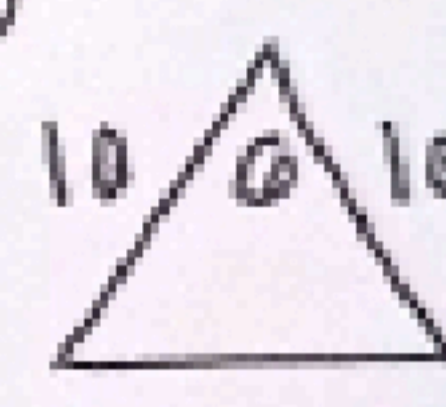
10) $y = x^{x^2}$
 $\ln y = x^2 \ln x$
 $(\ln y)' = x^2 \cdot \frac{1}{x} + \ln x (2x)$
 $= x + 2x \ln x$
 $= x(1 + 2 \ln x)$
 $= x(1 + \ln x^2)$ (c)

2) $f(x) = (x-1) \cos x$
 $f'(x) = (x-1)(-\sin x) + (\cos x)(1)$
 $= (1-x)(\sin x) + \cos x$ (a)

11)  $\frac{dx}{dt} = -26$
 $\frac{dy}{dt} ? , x=5$
 $y = \sqrt{144 + x^2} \Rightarrow \frac{dy}{dt} = \frac{2x \frac{dx}{dt}}{2\sqrt{144 + x^2}}$
 $\frac{dy}{dt} = \frac{7(5)(-26)}{7\sqrt{169}} = \frac{-5 \cdot 26}{13} = -10$ (b)

3) $v=0 \Rightarrow s(t) = 3t^2 - 9t + 6 = 0 / 3$
 $t^2 - 3t + 2 = 0$
 $(t-2)(t-1) = 0 \Rightarrow t = 1, 2$ (b)

4) $\frac{dy}{dx} = \frac{-\sqrt{2} \cos x}{\sin^2 x} \Rightarrow \frac{dy}{dx} \Big|_{\frac{\pi}{4}} = \frac{-\sqrt{2} \cdot \frac{1}{\sqrt{2}}}{\frac{1}{2}}$
 $= -\frac{1}{\frac{1}{2}} = -2$ (d)

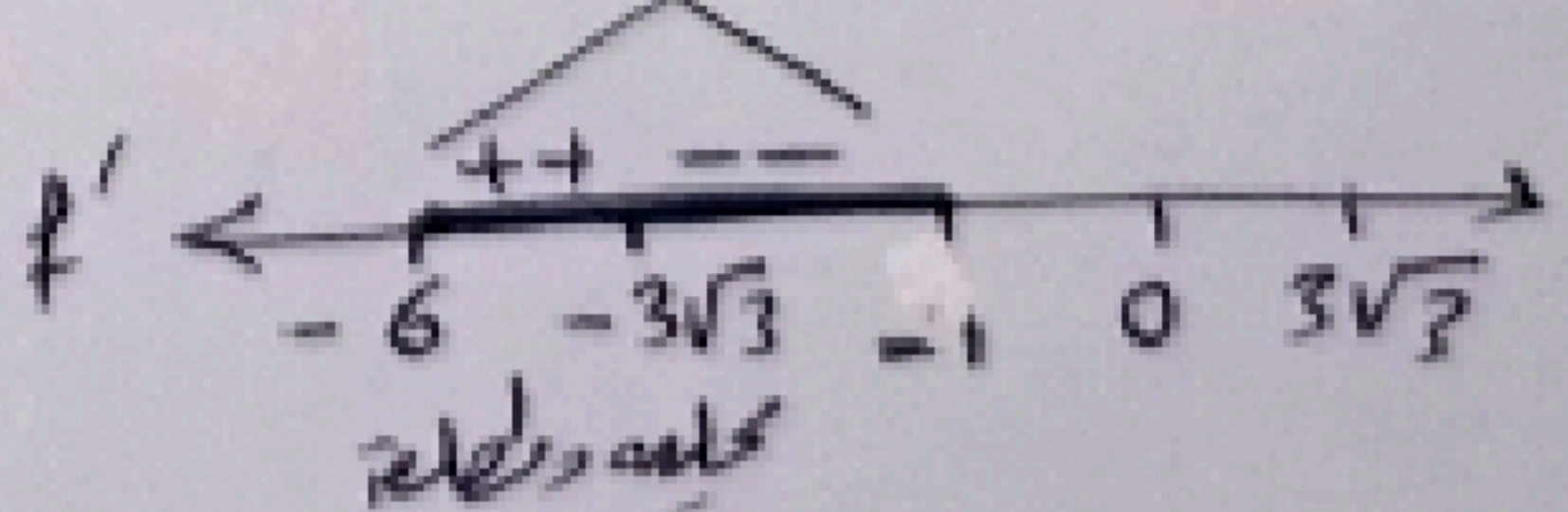
12)  $\frac{d\theta}{dt} = \frac{\pi}{60}$
 $\frac{dA}{dt} ? , \theta = \frac{\pi}{3}$
 $A = \frac{1}{2} (10)(10) \sin \theta = 50 \sin \theta$
 $\frac{dA}{dt} = 50 \cos \theta \frac{d\theta}{dt}$
 $\frac{dA}{dt} \Big|_{\theta = \frac{\pi}{3}} = 50 \left(\frac{1}{2}\right) \cdot \frac{\pi}{60} = \frac{5\pi}{12}$ (c)

6) $f(x) = \sqrt{\ln x} = \frac{1}{x} = \frac{1}{2x\sqrt{\ln x}}$
 $= \frac{1}{2x f(x)}$ (c)

7) $f'(x) = 0 \Rightarrow 3^{(x^2+1)} \cdot 2x \ln 3 = 0$
 $3^{x^2+1} = 0$ غير ممكن \vee $2 \ln 3 x = 0 \vee$ $x = 0$ (a)

13) $f'(x) = \frac{1}{3} \cdot \frac{9}{x^2} = \frac{x^2 - 27}{3x^2}$
 $x^2 - 27 = 0 \Rightarrow x = \pm \sqrt{27} = \pm 3\sqrt{3}$
 $3x^2 = 0 \Rightarrow x = 0$

8) $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2 \sec t \cdot \sec t \cdot \tan t}{2 \tan t \sec^3 x} = 1$ (d)

 $f' \leftarrow \begin{array}{c} ++ \\ - - \end{array}$
 $f(-3\sqrt{3}) = \frac{-3\sqrt{3}}{3} + \frac{9}{-3\sqrt{3}}$
 $= -\sqrt{3} - \frac{3}{\sqrt{3}}$
 $= -\sqrt{3} - \sqrt{3} = -2\sqrt{3}$ (b)

9) $xy y' = 2 \sin\left(\frac{\pi}{3}x\right) \cdot \frac{\pi}{3}$
 $y' = \frac{-\frac{\pi}{3} \sin\left(\frac{\pi}{3}x\right)}{y}$
 $y' = \frac{-\frac{\pi}{3} \frac{\sqrt{3}}{2}}{\frac{2}{\sqrt{3}}} = \frac{-\pi \sqrt{3}}{6}$ (c)

$y^2 = 2 \cos \frac{\pi}{3}$
 $= 2 \cdot \frac{1}{2}$
 $y^2 = 1$
 $y = 1$

5) $f' = 2 - \frac{2}{x^3} \Rightarrow f'' = \frac{2 \cdot 3x^2}{x^6}$
 $f'' = \frac{6}{x^4} \Rightarrow f''' = \frac{-6 \cdot 4x^3}{x^8} = \frac{-24}{x^5}$ (d)

١. نماذج هجرانه

③

2
3

② $f'(3) = \lim_{h \rightarrow 0} \frac{f(3+h) - f(3)}{h}$

$$= \lim_{h \rightarrow 0} \frac{\sqrt[3]{2(3+h)} - 6 + 4}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt[3]{6+2h} - 6}{h} = \lim_{h \rightarrow 0} \frac{(2h)^{\frac{1}{3}}}{h}$$

$$= 2^{\frac{1}{3}} \lim_{h \rightarrow 0} h^{-\frac{2}{3}} = \sqrt[3]{2} \lim_{h \rightarrow 0} \frac{1}{h^{\frac{2}{3}}}$$

$0^+ = \sqrt[3]{2} \lim_{h \rightarrow 0^+} \frac{1}{h^{\frac{2}{3}}} = \infty$
 $0^- = \sqrt[3]{2} \lim_{h \rightarrow 0^-} \frac{1}{h^{\frac{2}{3}}} = -\infty$

2
3

③ $5(\cot(\tan^2 \sqrt{2x^3+1}))$

* $-\csc^2(\tan^2 \sqrt{2x^3+1})$

* $2 \tan \sqrt{2x^3+1} \sec^2 \sqrt{2x^3+1}$

* $\frac{6x^2}{2\sqrt{2x^3+1}}$

3
3

④ $6yy' = 8x + xy' + y$

$6yy' - xy' = 8x + y$

$y'(6y - x) = 8x + y \Rightarrow y' = \frac{8x + y}{6y - x}$

$y' \Big|_{(-a, a)} = \frac{-8a + a}{6a - -a} = \frac{-7a}{7a} = -1 \neq$

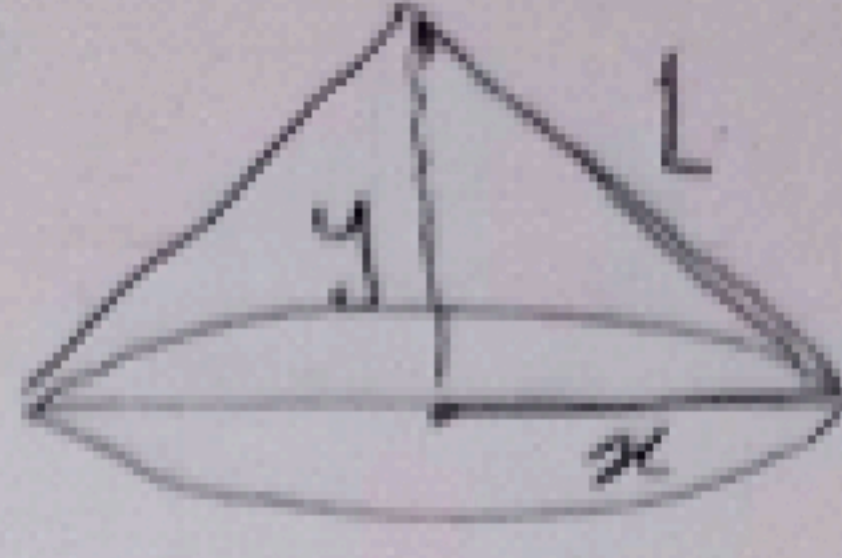
⑥

$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t}{3t^2 - 6t}$

$\frac{d^2y}{dx^2} = \frac{(3t^2 - 6t)(2 \frac{dt}{dx}) - (2t)(6t - 6)}{(3t^2 - 6t)^2}$

$\frac{d^3y}{dx^3} \Big|_{t=1} = \frac{(-3)(2 * \frac{1}{-3}) - (2)(0)(\frac{1}{-3})}{9} = \frac{2}{9}$

3
3

⑤  $\frac{dV}{dt} = 2$

$y = 2x$

$\pi x L = \text{مساحة سطح الجوانب للقنطرة}$

$A = x \pi L$

$A = x \pi \sqrt{5x^2}$

$A = x \pi \sqrt{5} x$

$A = \sqrt{5} \pi x^2$

$\frac{dA}{dt} = 2\sqrt{5} \pi x \frac{dx}{dt}$

$L^2 = y^2 + x^2$
 $L^2 = (2x)^2 + x^2$
 $L = \sqrt{5x^2}$

$V = \frac{1}{3} \pi x^2 y$

$V = \frac{1}{3} \pi x^2 \cdot 2x$

$V = \frac{2}{3} \pi x^3$

$\frac{dV}{dt} = \frac{2}{3} \pi \cdot 3x^2 \frac{dx}{dt}$

$2 = x \pi \cdot 36 \frac{dx}{dx} \Rightarrow \frac{dx}{dt} = \frac{1}{36\pi}$

$\therefore \frac{dA}{dt} = 2\sqrt{5} \pi (6) \cdot \frac{1}{36\pi}$

$= \frac{\sqrt{5}}{3}$

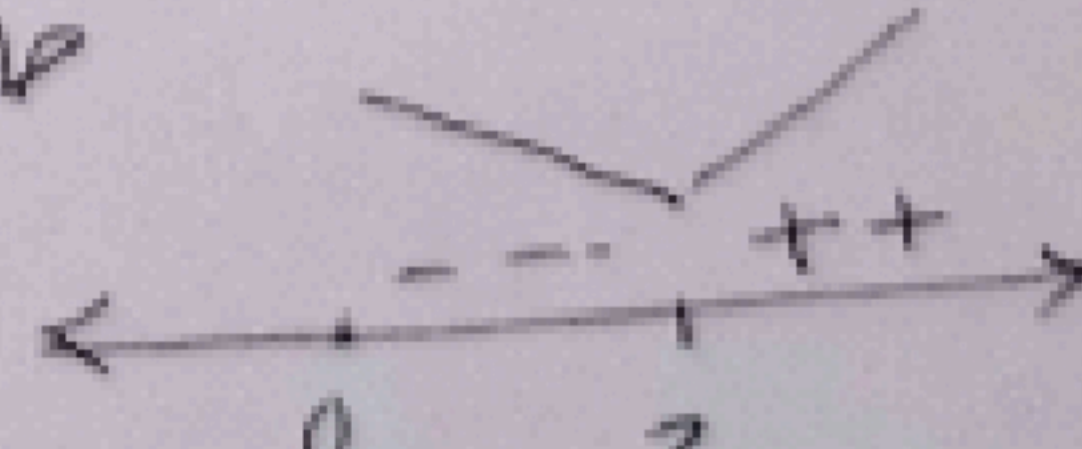
4
3

⑦ $f'(x) = x^2 - \frac{8}{x}$

$f(x) = \frac{x^3 - 8}{x}$

$x = 2 \leftarrow$ نقطة انحناء

$\frac{dy}{dx} x = 0 \leftarrow$ نقطة انحناء

$x > 0$ 

مناطق f : $(0, 2]$ متزايدة
 $[2, \infty)$ متنازلة

يوجد نقطة مفردة محلية دنيا عند $x = 2$ وهي $\frac{8}{3} - 8 \ln 2 = f(2)$